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Operate and Restraint Signals of a Transformer Differential Relay

OPERATE AND RESTRAINT SIGNALS OF A TRANSFORMER DIFFERENTIAL RELAY

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1. Introduction

The differential and restraining currents for transformer protective relaying can be formed by a digital relay in a number of ways. In the past the choice used to be driven by the limitations in the processing power of digital relays and a mindset resulting from available signal processing means in analog relays. Increase in processing power of contemporary microprocessor based relays makes possible to revisit the definitions of the differential and restraining quantities in search for an optimal solution.

The design choices for the differential signal include primarily two options: to form a phasor of the differential signal out of phasors of the input phase currents, or to form an instantaneous value of the differential signal out of raw samples of the input phase currents. This applies to fundamental frequency component (50 or 60Hz) as well as to the 2^{nd} and 5^{th} harmonics that are traditionally used to restrain a relay during inrush and overexcitation conditions, respectively.

The design choices for the restraining signal include the sum, the average and the maximum of the currents at all the connected circuits. A two-winding transformer is a special case and another optimal definition of the restraining current is available.

From the measuring perspective, the design choices for the differential and restraining quantities include the fundamental frequency phasor, the true RMS value and certain techniques emulating analog relays and based on raw samples.

The paper presents the possible design alternatives resulting from the combination of the aforementioned options and analyzes the resulting consequences for relay settings and performance.

A number of presented conclusions apply not only to a transformer differential relay but to low-impedance bus and line differential relays as well.

2. Differential and Restraining Currents

The differential protection principle is based upon comparison of currents at all the terminals of a protected element. The principle calls, however, for certain signal preprocessing when applied to a power transformer. A three-phase transformer apart from the transformation ratio, introduces certain angular displacement between the primary and secondary currents depending on the type of winding connections. In addition, the primary ratings of the CTs are limited to those of available standard values which creates an extra amplitude mismatch between the currents to be compared by the transformer differential relay.

Typically, electromechanical and static relays require the compensation for the transformer vector group and ratio to be performed outside the relay by appropriate connection of the main CTs (reversing the angular displacement) and the interposing CT (which can correct both the ratio and phase angle).

Some analog relays are equipped with taps enabling one to correct the ratio mismatch. As the differential current is just a linear combination of the phase currents from all the transformer terminals, some static relays compensate for the phase and amplitude mismatch using operational amplifiers and do not require interposing CTs nor any specific connection of the main CTs.

Digital relays perform the angular and ratio compensation numerically enabling reduction of wiring and the burden of the main CTs which, in turn, improves the operating conditions for the CTs and enhances performance of the entire protection system.

Section 3 and Appendix A address the issue of digital phase and amplitude compensation for all the practical winding connections of two- and three-winding transformers. Two basic design options are presented including sample-based and phasor-based compensation.

The biased differential principle calls for a "restraining current" as a comparison base for an operating (differential) current. The "sum of" approach is a traditional way of developing the restraining signal for a multi-circuit differential scheme, but other approaches are also possible that offer certain benefits over the traditional way. Section 4 addresses the issue of the restraining current.

Section 5 discusses various measuring algorithms, i.e. ways of converting raw samples of the input currents into a per-phase time-invariant quantity, for both differential and restraining signals. Special attention is paid to differences between the phasor magnitude and true RMS for heavily distorted waveforms.

Section 6 presents selected adaptive algorithms for differential and restraining quantities.

3. Differential Currents

Mathematically, the differential signal formed internally by a digital differential relay is a linear combination of the currents at all the terminals of a protected transformer. The way of forming the differential signal must meet the following requirements:

1. The differential current is zero under load and external fault conditions, and equals the fault current during internal faults.
2. The zero-sequence current does not affect the differential signal.

The first requirement is met by:

- matching the amplitudes of the currents at different terminals by multiplying by an appropriate matching factor, and
- matching the phases of the currents by "inverting" the transformer winding connection.

The second requirement that prevents the relay from overtripping during external ground faults due to discontinuity of the zero-sequence circuit in wye connected power transformers, is achieved either by:

- emulating the delta connection of the CTs for all the wye-connected windings of a protected transformer, or by
- calculating and subtracting the zero sequence current explicitly.

The operation of removing the zero sequence component is necessary also for a delta winding if there is an in-zone grounding transformer connected to the delta terminals or a cable is used to connect the transformer terminals to the breaker (significant in-zone zero sequence charging current).

As shown below, it is sufficient to analyze five basic winding connections to build the differential current formulae for any two- and three-winding three-phase transformer. These connections are: Yd30, Yy0, Dd0, Yz30 and Dz30 (here, y stands for a wye winding, d - for a delta winding and z - for a zig-zag winding, capital letters denote HV side, while numbers — a lagging phase shift in degrees).

Consider a three-phase three-winding transformer shown in Figure 1 with all its currents, including currents in the neutrals of wye and zig-zag windings, measured in one direction (either <u>into</u> – preferred – or <u>from</u> the transformer).

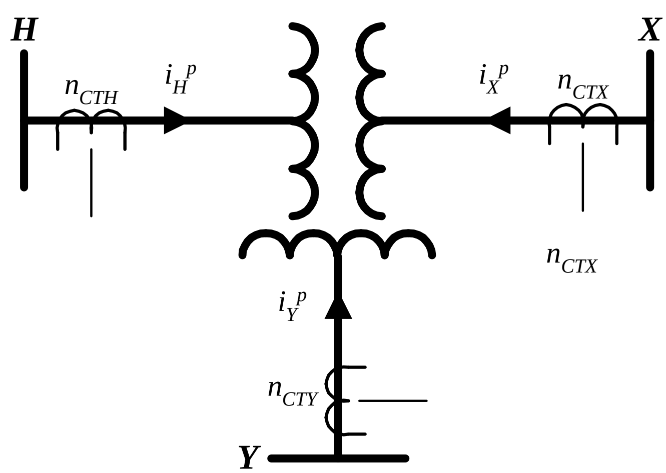

Figure 1. A three-winding transformer.

Let us denote:

V_H, V_X, V_Y	rated line-to-line voltages of the windings H, X and Y, respectively;
I_H, I_X, I_Y	rated currents, respectively;
n_{XH}, n_{YH}	current transformation ratios between the windings X and H, and Y and H, respectively:

$$n_{XH} = \frac{I_H}{I_X}, \quad n_{YH} = \frac{I_H}{I_Y} \tag{1}$$

n_{CTH}, n_{CTX}, n_{CTY}	transformation ratios of the main CTs, respectively.

3.1. Yd30 connected transformer

The winding connection of the Yd30 transformer is shown in Figure 2. Since the currents at the delta-connected winding do not contain the zero-sequence component, they may be taken as a base for the differential currents. From the figure we see that under load and external fault conditions the primary current $i_{XA}{}^{p}$ is balanced by the signal

$$\frac{1}{\sqrt{3}n_{XH}}\left(i_{HA}{}^{p} - i_{HC}{}^{p}\right)$$

which, as a difference of two line currents, does not include any zero-sequence component either. Thus, the primary differential current for the phase A is formed as:

$$i_{DA}{}^{p} = \frac{1}{\sqrt{3}n_{XH}}\left(i_{HA}{}^{p} - i_{HC}{}^{p}\right) + i_{XA}{}^{p} \tag{2}$$

or using the secondary currents:

$$i_{DA}{}^{p} = \frac{n_{CTH}}{\sqrt{3}n_{XH}}\left(i_{HA} - i_{HC}\right) + n_{CTX}\, i_{XA} \tag{3}$$

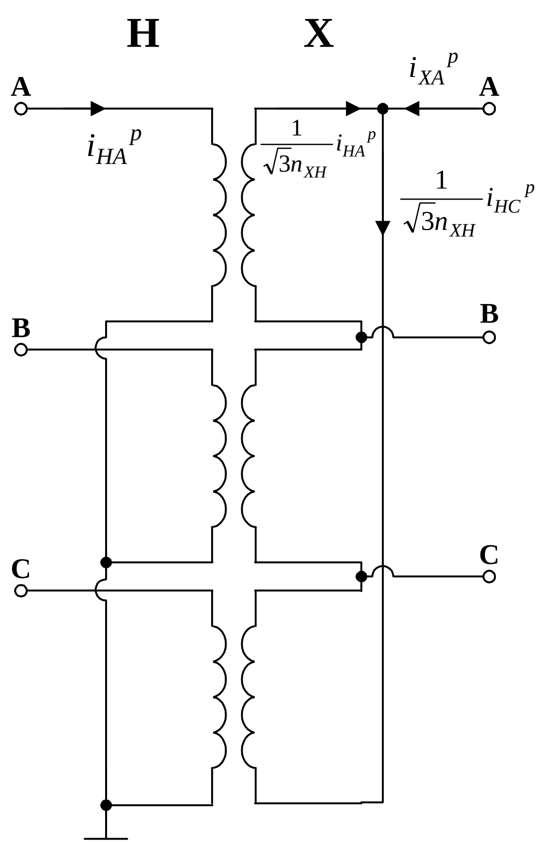

Figure 2. Yd30 connected transformer.

Relating the differential current to the secondary current of the CTs at the H side of a transformer (this will be consistently followed in this paper) yields:

$$i_{DA} = \frac{1}{\sqrt{3}}\left(i_{HA} - i_{HC}\right) + \frac{n_{XH}\, n_{CTX}}{n_{CTH}}\, i_{XA} \tag{4}$$

Let us now introduce a matching factor:

$$C_{XH} = \frac{n_{XH}\, n_{CTX}}{n_{CTH}} \tag{5}$$

Since the transformer ratio is a function of the rated voltages:

$$n_{XH} = \frac{V_X}{V_H} \tag{6}$$

thus:

$$C_{XH} = \frac{V_X\, n_{CTX}}{V_H\, n_{CTH}} \tag{7}$$

When the CTs are selected properly, the factor C_{XH} does not differ much from unity because the CTs invert approximately the transformation ratio of a protected transformer. Equation (7) is valid in all further considerations.

Certainly, in actual applications the factor $1/\sqrt{3}$ in (4) is accommodated by signal scaling subroutines. Here, however, it will be kept in order to maintain a clear base for the differential current (secondary amperes at the winding H).

The currents for the remaining phases B and C are derived from (4) by a simple subscript rotation and the complete set of equations for a Yd30 transformer becomes:

$$i_{DA} = \frac{1}{\sqrt{3}}\left(i_{HA} - i_{HC}\right) + C_{XH}\, i_{XA} \tag{8a}$$

$$i_{DB} = \frac{1}{\sqrt{3}}\left(i_{HB} - i_{HA}\right) + C_{XH}\, i_{XB} \tag{8b}$$

$$i_{DC} = \frac{1}{\sqrt{3}}\left(i_{HC} - i_{HB}\right) + C_{XH}\, i_{XC} \tag{8c}$$

Note that the above equations hold true for both raw samples of the currents and for their phasors and may be written in a compact matrix form as follows:

$$\begin{bmatrix} i_{DA} \\ i_{DB} \\ i_{DC} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} & C_{XH} & 0 & 0 \\ -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 & 0 & C_{XH} & 0 \\ 0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 & 0 & C_{XH} \end{bmatrix} \cdot \begin{bmatrix} i_{HA} \\ i_{HB} \\ i_{HC} \\ i_{XA} \\ i_{XB} \\ i_{XC} \end{bmatrix} \tag{9}$$

Generally, for any kind of a transformer we may write:

$$i_D = \mathbf{D} i \tag{10}$$

where:

i_{D} — vector of three differential currents,
i — vector of all the currents associated with a protected transformer,
D — constant transformation matrix.

The matrix ***D*** depends on the type of the transformer connections, ratios of the CTs and the rated voltages the transformer. Appendix A provides equations for various transformer connections.

The matrix ***D*** is a 3x6 matrix for two-winding, and 3x9 – for three-winding transformers. It displays a great deal of symmetry enabling efficient implementation.

The magnitude and angle compensation may be done on the raw samples or phasor levels as shown in Figure 3.

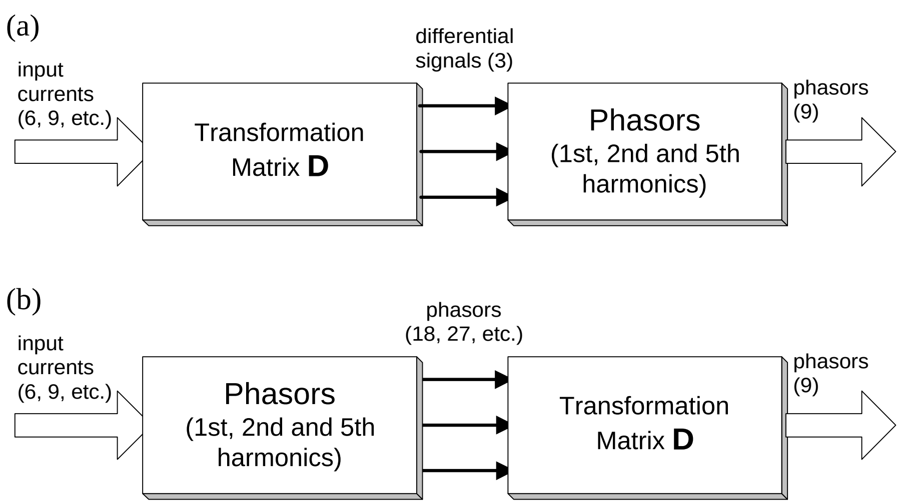

Figure 3. Phasors of differential currents formed from raw samples (a) and phasors (b).

Using raw samples of the input currents one obtains raw samples of the differential signals (Figure 3a). The latter enable application of wave-based protection principles (for example, the principle of a dwell-time during magnetizing inrush conditions). The fundamental frequency phasor and the 2nd and 5th harmonics are calculated next from the samples of the differential signal. A need to process, regardless of the number of windings, only three waveforms (i_A, i_B and i_C) into nine phasors (I_{DA}, I_{DB}, I_{DC}, I_{D2A}, …, I_{D5C}) is an advantage.

When using phasors of the input currents to form the differential signal one needs to calculate the fundamental frequency phasor as well as the 2nd and 5th harmonics first (Figure 3b). This totals 3 (phases) x 3 (quantities) x 2 (windings) = 18 phasors for two-winding, and 27 phasors for three winding transformers. Next, the calculated phasors are transformed into the differential quantities using the matrix ***D***.

3.2. Three-winding transformers

The following algorithm may be used to build the differential currents for three-winding transformers:

1. Consider a pair of windings, say H and X and obtain a portion of the formula for the differential current.
2. Consider another pair of windings, say H and Y (or X and Y, appropriately) and obtain the completing portion of the formula for the differential current.
3. Consider the remaining pair X and Y or (H and Y, appropriately) for the final check.

Let us use the Ydd 0/150/330 transformer as an example.

First, we consider the Yd150 connection (HX) and from Table A.1 we obtain:

$$i_{DA}' = \frac{1}{\sqrt{3}}\left(i_{HB} - i_{HA}\right) + C_{XH}\, i_{XA} \tag{11}$$

Next, we consider the Yd330 connection (HY) and we get:

$$i_{DA}'' = \frac{1}{\sqrt{3}}\left(i_{HA} - i_{HB}\right) + C_{YH}\, i_{YA} \tag{12}$$

Combining (11) and (12) yields the formula:

$$i_{DA} = \frac{1}{\sqrt{3}}\left(i_{HB} - i_{HA}\right) + C_{XH}\, i_{XA} + C_{YH}\left(-i_{YA}\right) \tag{13}$$

For the final check we consider the Dd180 connection (XY) and compare the last two agents in (13) with Table A.2. The formulae for the two remaining phases B and C are, certainly, regenerated from (13) by appropriate rotation of the phase indices.

4. Restraining Currents

For stability during external faults and load conditions with ratio mismatch and/or saturation of the CTs, a differential relay uses a restraining quantity as a reference for the differential signal.

Traditionally, the following "sum of" formula has been used for the restraining signal for a *n*-circuit differential relay:

$$i_R = |i_1| + |i_2| + |i_3| + \ldots + |i_n| \tag{14}$$

The alternatives include:

$$i_R = \frac{1}{2}|i_1 - i_2| \qquad \text{(for two-winding transformers only)} \tag{15}$$

$$i_R = \frac{1}{n}\left(|i_1| + |i_2| + |i_3| + \ldots + |i_n|\right) \qquad (n \text{ - number of } \underline{\text{actually}} \text{ connected circuits}) \tag{16}$$

$$i_R = Max\left(|i_1|, |i_2|, |i_3|, \ldots, |i_n|\right) \tag{17}$$

The above options are discussed below starting with a two-winding transformer as a special case.

4.1. Two-winding transformers

Consider a simple circuit shown in Figure 4 that neglects the ratio and vector group a transformer. The differential current is formed in such a case as (note the direction of the currents):

$$i_D = i_H + i_X \tag{18}$$

The through current used as the restraining signal for two-winding transformers should be calculated as:

$$i_R = \frac{1}{2}(i_H - i_X) \tag{19}$$

in order to reflect an average current at both the sides of a transformer for load and external fault situations.

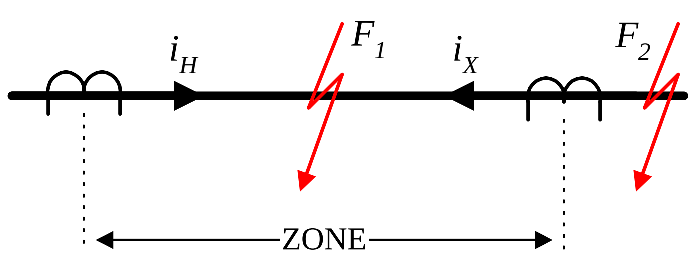

Figure 4. Simple circuit for analysis of two-winding transformers.

Adopting the definitions (18) and (19) we find the following important relations:

- for external faults (F_2) neglecting the load current we obtain i_H = -i_X and therefore:

$$|i_D| = |i_H + i_X| = 0 \tag{20a}$$

$$|i_R| = \frac{1}{2}|i_H - i_X| = |i_H| = |i_X| = |i_{F2}| >> |i_D| \tag{20b}$$

- for internal faults (F_1) i_H and i_X are almost in phase and:

$$|i_D| = |i_H + i_X| = |i_{F1}| > \max(|i_H|, |i_X|) \tag{21a}$$

$$|i_R| = \frac{1}{2}|i_H - i_X| < \max(|i_H|, |i_X|) < |i_D| \tag{21b}$$

Let us compare the above approach with alternatives (14), (16) and (17).

Note, that for internal faults:

$$\max(|i_H|, |i_X|) \geq \frac{1}{2}(|i_H| + |i_X|) \geq \frac{1}{2}|i_H - i_X| \tag{22}$$

Relation (22) mean that the definition (15) provides the smallest restraint and thus enhances the relay sensitivity.

For external faults the restraint (15) is not smaller than the other alternatives. Thus, (15) does not jeopardize relay security.

The above proves that the formula (15) is optimal for two-winding transformers. Figure 5 offers a graphical explanation.

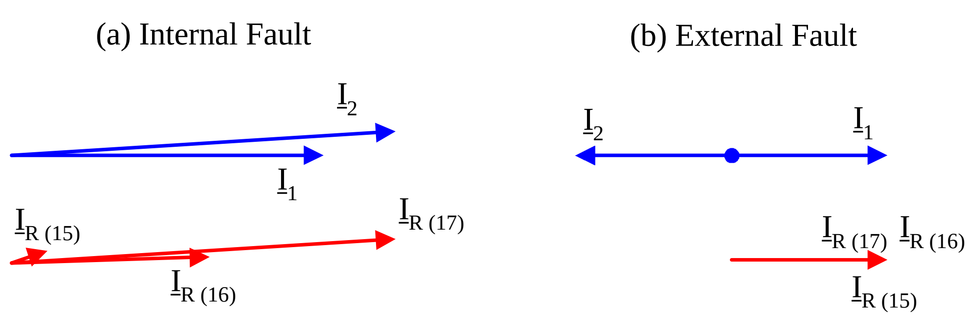

Figure 5. Formula (15) produces the smallest restraint for internal faults (a) and same restraint for external faults (b).

A three-phase transformer introduces a ratio mismatch and angular displacement between its primaries and secondaries. The restraining currents defined as (14), (16) or (17) may neglect the phase displacement since they refer directly to the amplitudes. The restraining current defined by (15) must, however, account for both the ratios and the angular displacement.

Note that the restraining current (19) may be directly derived from the differential current (18) by changing the sign and adding the factor $\frac{1}{2}$. Therefore, for two-winding transformers appropriately transformed differential current equations may be used directly for the restraining signals. For example, for the Yd30 connected transformer equation (8) yields:

$$i_{RA} = \frac{1}{2}\left[\frac{1}{\sqrt{3}}\left(i_{HA} - i_{HC}\right) - C_{XH}\, i_{XA}\right] \tag{23a}$$

$$i_{RB} = \frac{1}{2}\left[\frac{1}{\sqrt{3}}\left(i_{HB} - i_{HA}\right) - C_{XH}\, i_{XB}\right] \tag{23b}$$

$$i_{RC} = \frac{1}{2}\left[\frac{1}{\sqrt{3}}\left(i_{HC} - i_{HB}\right) - C_{XH}\, i_{XC}\right] \tag{23c}$$

Generally, using (15) for any two-winding three-phase transformer one may write:

$$i_R = \mathbf{R} i \tag{24}$$

where:

i_{R} — vector of three restraining currents,
R — constant transformation matrix.

The matrix ***R*** depends on the type of the transformer connections, ratios of the CTs and the rated voltages of the transformer.

Equations (24) are valid for both raw samples and phasors of the input currents.

4.2. Multi-circuit elements

Unfortunately, the optimal formula for the restraining signal for two-winding transformers (15) cannot be extended to multi-circuit elements to be protected if more than one source of power is connected to the element.

The design choices in this case are given by equations (14), (16) and (17).

For illustration of the differences between the above alternatives Table 1 delivers a simple numerical example for a 6-circuit element (busbar) with 2 circuits disconnected.

The first two rows in the table are actually equivalent; the relation between:

$i_R = |i_1| + |i_2| + |i_3| + ... + |i_n|$ versus $i_D = |i_1 + i_2 + i_3 + ... + i_n|$ and

$i_R = \frac{1}{n}\left(|i_1| + |i_2| + |i_3| + ... + |i_n|\right)$ versus $i_D = |i_1 + i_2 + i_3 + ... + i_n|$

is the same except a constant of n. The difference can be accommodated by the slope setting.

Depending on the approach taken for the restraining current the slope and break-point settings of the operating characteristic will differ. Table 2 illustrates that by analyzing the maximum possible ratio between the differential and restraining signals for internal faults and the maximum possible ratio between the restraining and fault currents for external faults.

Table 1. Numerical example for a 6-circuit element
(* — *n* is fixed (*n*=6), ** — *n* is adaptable (*n*=4); an external fault).

I_1	I_2	I_3	I_4	I_5	I_6	Formula	I_R
20	5	5	10	0	0	$i_R = \lvert i_1 \rvert + \lvert i_2 \rvert + \lvert i_3 \rvert + ... + \lvert i_n \rvert$	40
20	5	5	10	0	0	$i_R = \frac{1}{n}\left(\lvert i_1 \rvert + \lvert i_2 \rvert + \lvert i_3 \rvert + ... + \lvert i_n \rvert\right)^*$	6.67
20	5	5	10	0	0	$i_R = \frac{1}{n}\left(\lvert i_1 \rvert + \lvert i_2 \rvert + \lvert i_3 \rvert + ... + \lvert i_n \rvert\right)^{**}$	10
20	5	5	10	0	0	$i_R = Max\left(\lvert i_1 \rvert, \lvert i_2 \rvert, \lvert i_3 \rvert, ..., \lvert i_n \rvert\right)$	20

Table 2. Analysis of the amount of restraint during internal and external faults
(* — *n* is fixed, ** — *n* is adaptable).

Formula	Internal Faults: Maximum I_D / I_R	External Faults: Maximum I_R / I_{FAULT}
$i_R = \lvert i_1 \rvert + \lvert i_2 \rvert + \lvert i_3 \rvert + ... + \lvert i_n \rvert$	100%	200%
$i_R = \frac{1}{n}\left(\lvert i_1 \rvert + \lvert i_2 \rvert + \lvert i_3 \rvert + ... + \lvert i_n \rvert\right)^*$	$n \cdot 100\%$	$\frac{200\%}{n}$
$i_R = \frac{1}{n}\left(\lvert i_1 \rvert + \lvert i_2 \rvert + \lvert i_3 \rvert + ... + \lvert i_n \rvert\right)^{**}$	$n \cdot 100\%$	$\frac{200\%}{n}$
$i_R = Max\left(\lvert i_1 \rvert, \lvert i_2 \rvert, \lvert i_3 \rvert, ..., \lvert i_n \rvert\right)$	$n \cdot 100\%$	100%

4.3. Restraining currents for three-winding transformers

When forming the restraining current for a three-winding transformer, the two following cases should be distinguished:

1. Only one of the windings is connected to the power source.
2. More than one winding is connected to the source.

In the first case, the transformer may be still considered as a two-winding unit. The load windings, say X and Y, are grouped together by adding their currents in order to obtain the total load current.

The current in the supplying winding, say H, and the load current satisfy the principle of the through current, and therefore, the restraining signal may be formed as for two-winding transformers:

$$i_R = \frac{1}{2}\left[i_H - \left(i_X + i_Y\right)\right] \tag{25}$$

Equation (25) takes its detailed form for a particular transformer and accounting for the amplitude and phase mismatch. Again, let us use the Ydd 0/150/330 as an example. From (13) we obtain:

$$i_{RA} = \frac{1}{2}\left[\frac{1}{\sqrt{3}}(i_{HB} - i_{HA}) - [C_{XH}\, i_{XA} + C_{YH}\,(-i_{YA})] \right] \tag{26}$$

As previously, the above value is in secondary amperes at the winding H. Either samples or phasors can be used.

This idea, however, cannot be extended to three-winding transformers with more than one winding connected to a power source.

Therefore, for three-winding transformers the restraining current is generally formed using (14), (16) or (17) as illustrated below for the Ydd 0/150/330 transformer.

For example, equation (14) yields:

$$|\, i_{RA}| = \left(|\, i_{HA}| + C_{XH}\,|\, i_{XA}| + C_{YH}\,|\, i_{YA}|\right) \tag{27a}$$

But also, the formula for the differential current may be reused here. For the Ydd 0/150/330 transformer, equation (13) yields:

$$|i_{RA}| = \left[\frac{1}{\sqrt{3}}|i_{HB} - i_{HA}| + C_{XH}\,|i_{XA}| + C_{YH}\,|i_{YA}| \right] \tag{27b}$$

5. Measuring Algorithms

5.1. Differential current

Typically, a fundamental frequency (60Hz) phasor is used as the operating quantity. A number of phasor estimators have been proposed in the literature. The dominating approach is, however, to use either a full- or half-cycle Fourier algorithm with pre-filtering aimed at rejecting the dc component.

To extract the harmonics from the differential current, typically the Fourier algorithm is used as well. Magnitudes alone of the 2nd and 5th harmonics are used to restrain the relay during inrush and overexcitation conditions, respectively. However, if the differential signal is formed out of phasors, then both the magnitude and angle of the harmonics of the input currents must be measured to form the differential 2nd and 5th harmonics.

5.2. Restraining current

There are more design choices with respect to the restraining quantity. The three alternatives discussed here are:

1. phasor magnitude (50 or 60Hz component).
2. one-cycle true RMS.

3. a sample-based approach such as:

$$I_{R(k)} = \max\left(\left|i_{R(k)}\right|, \mathrm{b} \cdot I_{R(k-1)}\right) \tag{28}$$

where b is an arbitrary number less than but close to 1.00;
k is a "protection pass" index (sample number).

The true RMS as a restraining quantity has the following advantages over the phasor magnitude (see Figure 6 for illustration):

(a) When the waveform gets distorted due to saturation of the CT, the true RMS is always larger than the magnitude of the phasor.

(b) The dc component contributes to the true RMS value. The true RMS is initially significantly larger than the phasor even for sinusoidal signals providing more security during transients.

(c) The true RMS as an estimator is not affected by the current reversal phenomenon. When the pre-fault and fault currents differ significantly as to the phase angle (current reversal), then the Fourier algorithm tends to deliver momentarily a magnitude lower than the pre-fault value (see Figure 7). This may case a relay to maloperate during external faults.

Let us analyze more precisely the effect of CT saturation on both the true RMS and the full-cycle-Fourier-estimated phasor magnitude.

Assuming symmetrical CT saturation (Figure 8a) we calculate the fraction of the actual ac magnitude seen as a true RMS as the following function of the angle to symmetrical saturation, a:

$$\frac{I_{RMS}}{I_{ACTUAL}} = \sqrt{\frac{1}{\mathrm{p}}\left(\mathrm{a} - \frac{1}{2}\sin(2\mathrm{a})\right)} \tag{29}$$

and the fraction of the actual ac magnitude seen as a phasor as the following function of the angle to symmetrical saturation:

$$\frac{I_{FOURIER}}{I_{ACTUAL}} = \frac{1}{2\mathrm{p}}\left|\frac{1}{2}\left(1-\exp(2j\mathrm{a})\right) + j\mathrm{a}\right| \tag{30}$$

Figure 9 provides the plot of the two functions. For example, if the CT saturates at 40 degrees (i.e. after 1.85 msec on a 60Hz system), the phasor magnitude will read only about 15% of the actual current magnitude. The true RMS will read about 25% of the actual magnitude.

For any depth of saturation, the true RMS delivers a higher value. Figure 10 plots the ratio between the true RMS and the phasor magnitude for various saturation angles. For very fast saturation, the true RMS could be 2 or more times as high as the phasor magnitude.

Assuming asymmetrical CT saturation (Figure 8b), we calculate the fraction of the actual ac magnitude seen as a true RMS as the following function of the angle to asymmetrical saturation:

$$\frac{I_{RMS}}{I_{ACTUAL}} = \sqrt{\frac{1}{\pi}\left(\frac{3}{2}\alpha + 2\sin(\alpha)\left(\frac{1}{4}\cos(\alpha)-1\right)\right)} \tag{31}$$

and the fraction of the actual ac magnitude seen as a phasor magnitude as the following function of the angle to asymmetrical saturation:

$$\frac{I_{FOURIER}}{I_{ACTUAL}} = \frac{1}{\pi}\left| j\left(\frac{3}{4} - \exp(j\alpha) + \frac{1}{4}\exp(j2\alpha)\right) - \frac{\alpha}{2}\right| \tag{32}$$

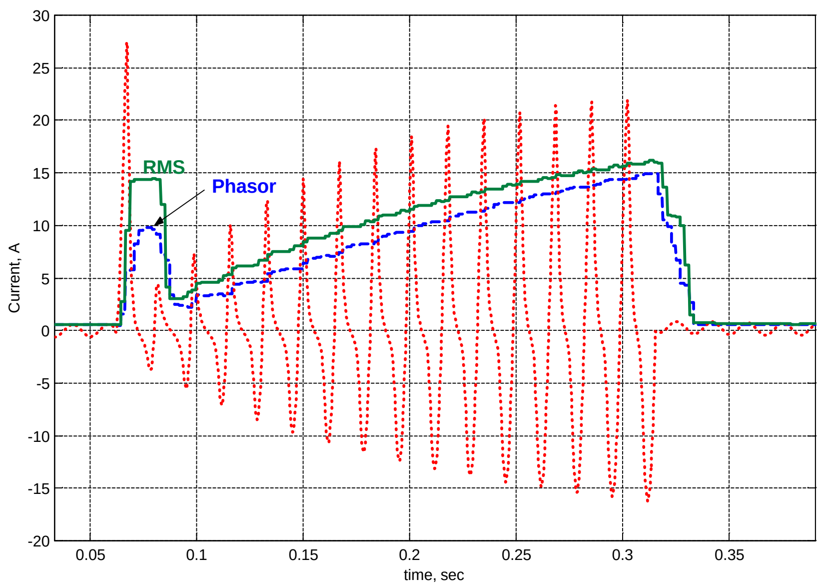

Figure 6. Sample current from a saturated CT — true RMS vs. phasor magnitude.

Figure 11 compares the true RMS and phasor magnitude for asymmetrical CT saturation. For example, if the saturation occurs at 180 degrees (i.e. after half of a power cycle), then the phasor magnitude reads about 80% of the actual ac magnitude, while the true RMS reads about 120% — more than the actual value due to the dc component.

Figure 12 presents the ratio between the true RMS and the phasor magnitude for an asymmetrically saturated waveform. For quick saturation, the RMS could be 3 or more times larger than the phasor magnitude.

The relations shown in Figures 10 and 12 illustrate certain danger in using the phasor magnitude for the differential, and the true RMS for the restraining signals — If only one feeder (bus) or one winding (transformer) feeds an internal fault and the CTs saturate quickly, there may be not enough differential signal for the biased differential element to operate.

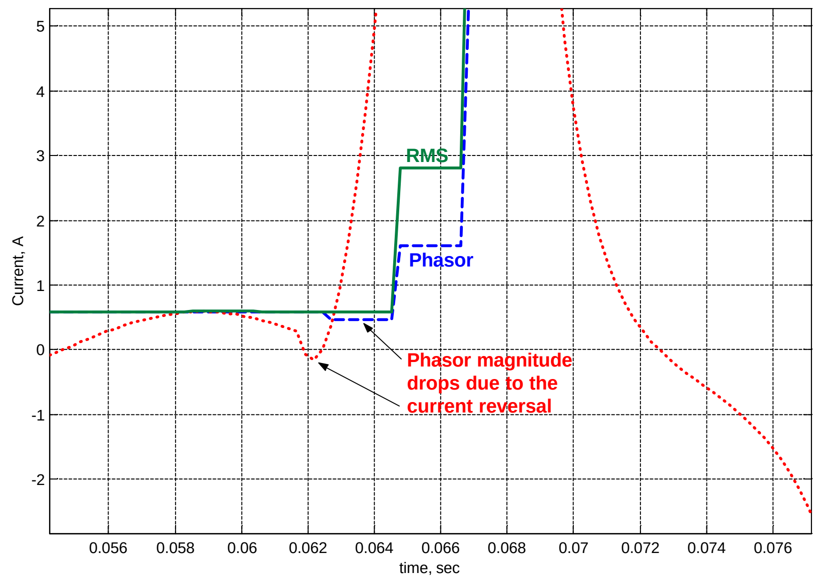

Figure 7. Magnification of Figure 6 — illustration of the current reversal phenomenon.

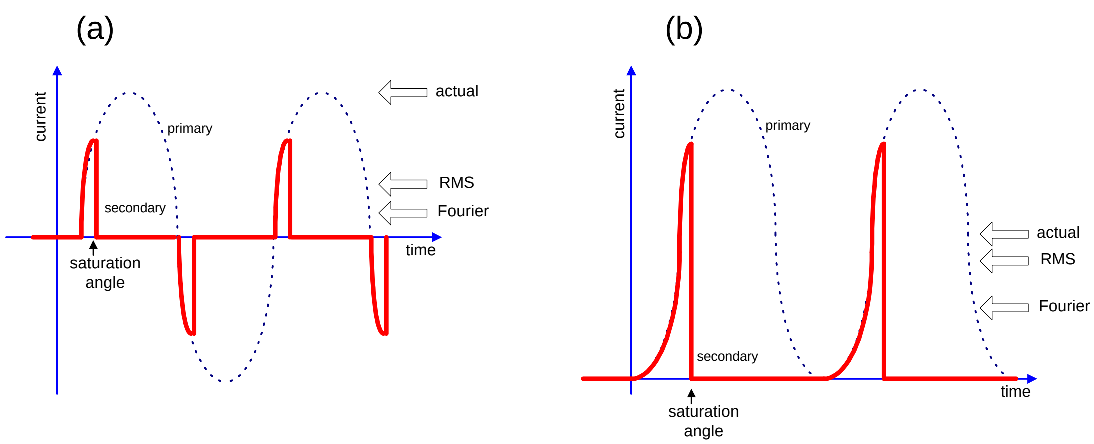

Figure 8. Assumed signal models for symmetrical (a) and asymmetrical (b) CT saturation.

Similarly, the alternative (28) provides a lot of restraining during heavy saturation and in the presence of the dc component. Figure 13 illustrates this. However, this approach may lead to missing relay operation as the phasor of the differential current is underestimated during heavy saturation, while the restraining current calculated as (28) may be significantly overestimated due to the dc component.

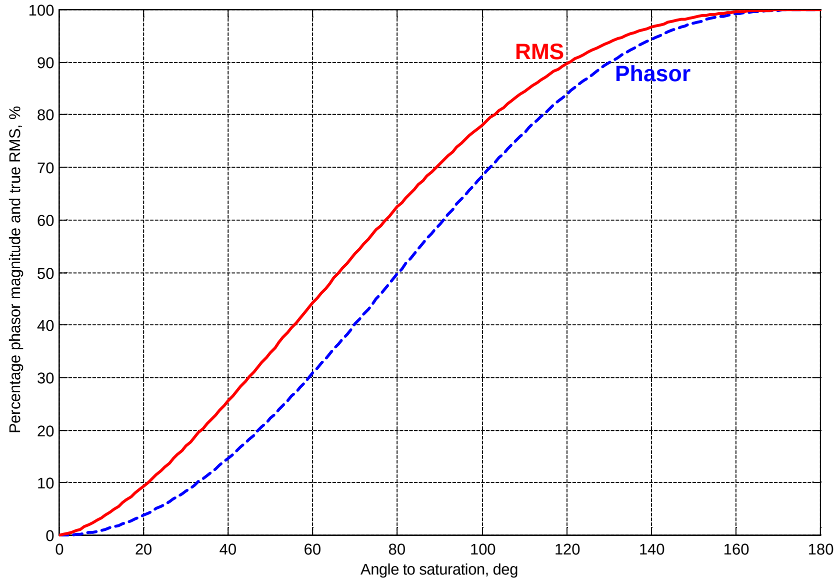

Figure 9. Percentage of the ac actual magnitude as seen by the true RMS and Fourier algorithms vs. the angle to symmetrical saturation (180deg = no saturation).

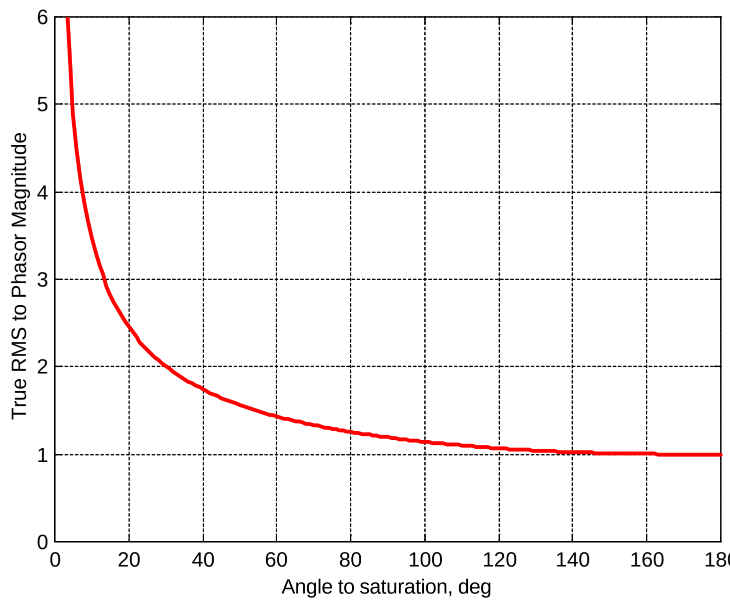

Figure 10. Ratio between the true RMS and phasor magnitude vs. the angle to symmetrical saturation.

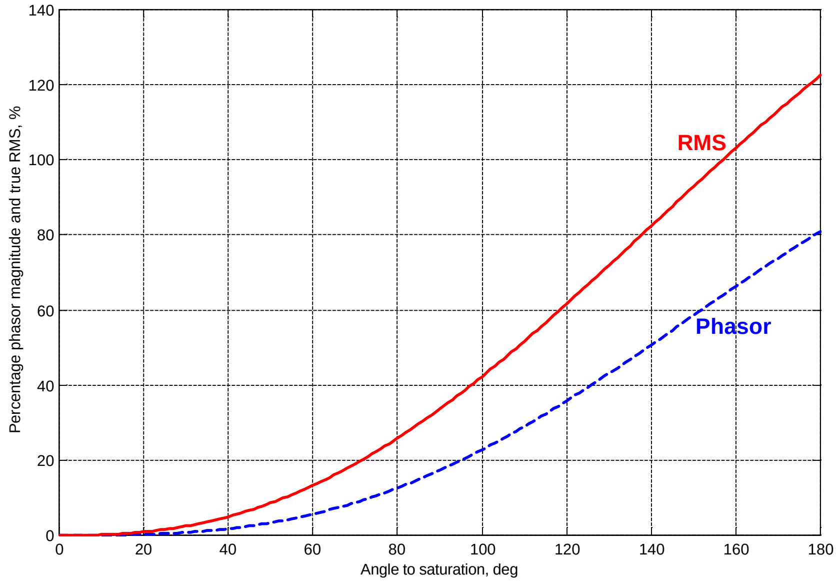

Figure 11. Percentage of the actual ac magnitude as seen by the true RMS and Fourier algorithms vs. the angle to asymmetrical saturation (180deg = saturation after half of a power cycle).

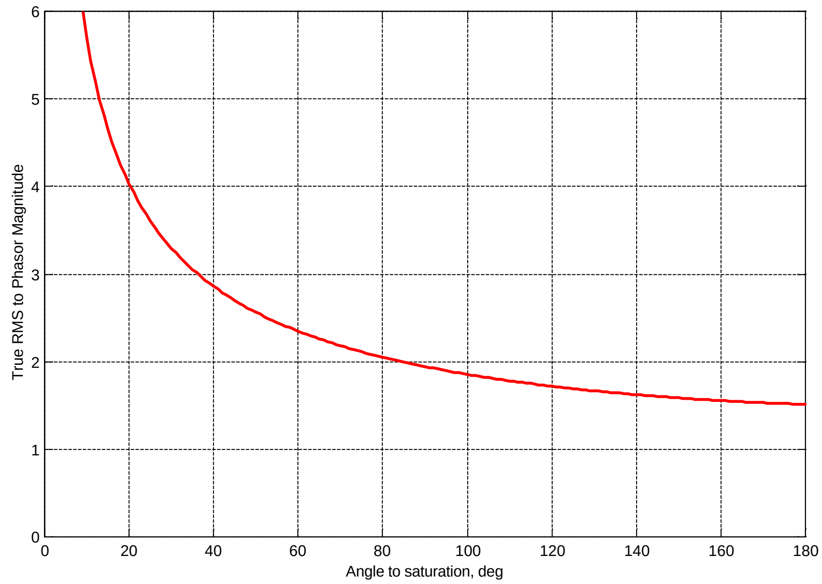

Figure 12. Ratio between the true RMS and phasor magnitude as vs. the angle to asymmetrical saturation.

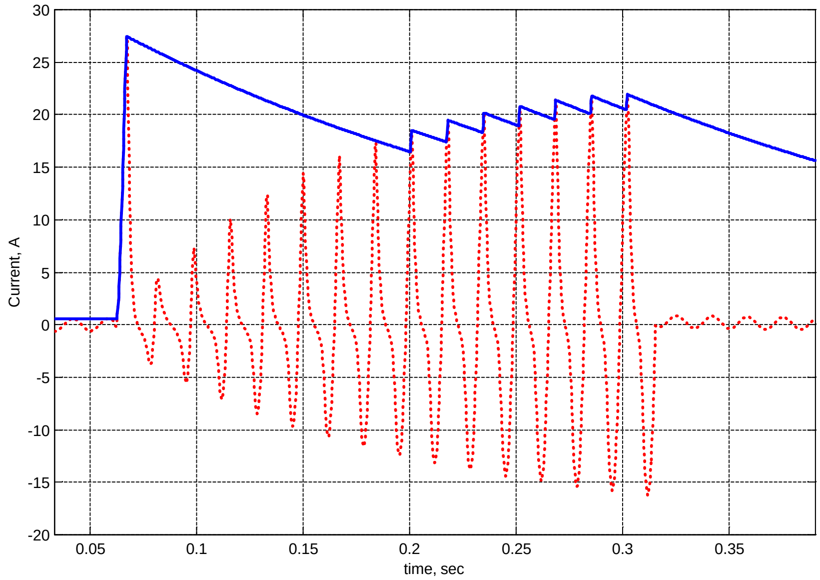

Figure 13. Sample current from a saturated CT — equation (28) used for the restraining quantity.

6. Adaptive Algorithms

6.1. Adaptive restraining current for three-winding transformers

Consider a three-winding transformer as in Figure 1. Assume that one of its windings, say Y, is disconnected from the outside system. With its one winding disconnected, a three-winding transformer may be treated as a two-winding unit with the restraining current calculated by the formula:

$$|i_R| = \frac{1}{2}|i_H - i_X| \tag{33}$$

This reasoning stands behind the adaptive approach to the restraining current in three-winding transformers. Assume that the positions of the Circuit Breakers (CBs) in all the windings of a transformer are available as the all-or-nothing variables s_H, s_X and s_Y ($s = 1$ — CB closed, $s = 0$ — CB open).

A simple adaptive algorithm for the restraining signal would be as follows:

$$if \;\; s_H = 0 \;\; then \;\; |i_R| = \frac{1}{2}|i_X - i_Y| \;\; else \tag{34a}$$

$$if \;\; s_X = 0 \;\; then \;\; |i_R| = \frac{1}{2}|i_H - i_Y| \;\; else \tag{34b}$$

$$if \;\; s_Y = 0 \;\; then \;\; |i_R| = \frac{1}{2}|i_H - i_X| \;\; else \tag{34c}$$

$$\left| i_R \right| = \max\left(\left| i_H \right|, \left| i_X \right|, \left| i_Y \right| \right) \tag{34d}$$

The first case (34a) means that no infeed is possible from the winding H, thus regardless of the positions of the remaining CBs, the restraining current may be formed as for a two-winding transformer having the windings X and Y.

The last case (34d) defines the general situation with all the CBs closed.

6.2. Adaptive differential current (compensation for the tap changer)

Consider a case of an on-load tap changer installed on a protected transformer. When the taps move from the position for which the transformer ratio was considered when setting the relay (usually the middle position), a amplitude mismatch occurs resulting in a false differential current.

Assuming the actual tap position, *p*, is available, the relay can adapt itself to cover the tap changes and keep the settings low, thus the sensitivity high.

The actual transformer ratio (6) under the tap position *p* equals:

$$n_{XH} = \frac{V_X (1 + p\Delta)}{V_H} \tag{35}$$

where: Δ is a p.u. tap step.

Thus, the correcting factor C_{XH} (7) for the differential current becomes:

$$C_{XH(p)} = \frac{V_X (1 + p\Delta) n_{CTX}}{V_H n_{CTH}} \tag{36a}$$

which may be rewritten as:

$$C_{XH(p)} = C_{XH(0)} (1 + p\Delta) \tag{36b}$$

and means that the ratio factor C_{XH} is compensated for the tap position by a simple linear function.

6.3. Adaptive differential current (compensation for small ratio/angle mismatch)

Consider a case of a non-zero current appearing during normal operation of a transformer and caused by slight differences among the installed CTs, aging, noise, etc.

In such a case, the ratio factor C_{XH} may be adjusted adaptively by the relay.

Note that the algorithm proposed below should be executed periodically only after making sure the transformer is in the mode of normal operation.

Under normal operation, the differential current given as:

$$i_D = i_H + C_{XH} i_X \tag{37}$$

should be zero. Since i_H and i_X are measured, we may consider C_{XH} as an unknown and minimize the average square error:

$$\sum_{l=0}^{M}\left(i_{H(k-l)} + C_{XH} i_{X(k-l)}\right)^2 \rightarrow \min \qquad (38)$$

where: k — present sampling instant;
M — assumed time horizon in samples (few or tens of power cycles).

When solved, the least square error problem (38) yields the following estimator for the matching factor:

$$\hat{C}_{XH} = -\frac{\sum_{l=0}^{M} i_{H(k-l)} i_{X(k-l)}}{\sum_{l=0}^{M} i_{X(k-l)}{}^2} \qquad (39)$$

The above procedure is numerically efficient and accurate. Nevertheless, extra security checks are recommended prior to using the estimate $\hat{C}_{XH}$ as the effective value of the matching factor.

Unfortunately, the presented approach cannot be directly extended to three-winding transformers. Because as many as two matching factors are involved, there is an infinite number of pairs that reduce the unbalance signal to zero. However, the relay sensitivity may be still increased by tuning one matching factor while leaving the other at its rated value.

7. Conclusions

A number of approaches to differential and restraining currents for transformer protection have been discussed. This includes quantities formed from phasors and from raw samples, true RMS and phasors as well as adaptive techniques.

Some of the design options are irrelevant for the end user while others may affect the settings as well as the performance of the transformer differential relay.

The phasors of a differential signal belong to the first category. Since both an operation of forming the differential signal from input currents and a phasor measuring algorithm are linear operations, there is absolutely no difference between the differential phasors formed out of samples or out of input phasors. The same applies to phasors of the 2nd and 5th harmonics as long as they are used as differential harmonics. The waveform-based approach has, however, an advantage of producing a waveform of the differential current and applying additional wave-based principles to enhance the performance.

The restraining currents belong to the second category. Depending on the applied definition, the relations between the fault current and the restraining quantity may differ significantly. This must be taken into account when setting the relay. The relay performance will be affected by the choice as well.

The decision to use the true one-cycle RMS versus the phasor magnitude for the restraining signal is one of such performance issues. The true RMS provides more restraining when the CTs saturate which is good for through fault stability, but it may deteriorate sensitivity as well.

Adaptive techniques can be applied to both the differential and restraining currents. Ratio and angular errors of the CTs as well as errors caused by the operation of a tap changer can be accommodated by adaptive calculation of differential currents. Increased sensitivity for three-winding transformers can be achieved without jeopardizing security by the adaptive approach to restraining currents.

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Biographies

Bogdan Kasztenny received his M.Sc. (89) and Ph.D. (92) degrees (both with honors) from the Wroclaw University of Technology (WUT), Poland. In 1989 he joined the Department of Electrical Engineering of WUT. In 1994 he was with Southern Illinois University in Carbondale as a Visiting Assistant Professor. From 1994 till 1997 he was involved in applied research for Asea Brown Boveri in the area of transformer and series compensated line protection. During the academic year 1997/98 Dr.Kasztenny was with Texas A&M University as a Senior Fulbright Fellow, and then, till 1999 - as a Visiting Assistant Professor. Currently, Dr.Kasztenny works for General Electric Company as a Senior Application/Invention Engineer. Dr.Kasztenny is a Senior Member of IEEE, holds 2 patents, and has published more than 90 technical papers.

Ara Kulidjian received the B.A.Sc. degree in electrical engineering from the University of Toronto, Canada, in 1991. He then joined GE Power Management where he has been involved in system and algorithm design of protection and control systems. He is a Registered Professional Engineer in the Province of Ontario and a member of the Signal Processing Society of the IEEE.

Bruce Campbell graduated in Electrical Technology from the Northern Alberta Institute of Technology in 1964. He has been involved in the design, commissioning and startup of high voltage electrical equipment in North America, the Caribbean, Africa, the Middle East and Southeast Asia. He is presently the chief application engineer for GE Power Management, involved in conceptual and scheme design for digital protective relays, and consulting on power system protection. He is a member of PES of the IEEE.

Marzio Pozzuoli graduated from Ryerson Polytechnical Institute, Toronto, Ontario Canada, in 1987 with a Bachelor of Electrical Engineering Technology specializing in control systems. He then worked for Johnson Controls designing industrial automation systems. He was involved in the design of Partial Discharge Analysis systems for large rotating electric machinery with FES International. In 1994 he joined General Electric – Power Management and is the Technology Manager responsible for the engineering and development of new products.

Appendix A. Differential Currents for Various Transformers

Let us consider the Yd90 transformer. It may be analyzed as the Yd30 unit with its terminals rotated as follows (for ABC system phase rotation):

H-side	X-side
$A \rightarrow C$	$A \rightarrow B$
$B \rightarrow B$	$B \rightarrow A$
$C \rightarrow A$	$C \rightarrow C$

Consequently, the differential currents for the Yd90 transformer are derived from (8) using the aforementioned rotation of the phase subscripts:

$$i_{DA} = \frac{1}{\sqrt{3}}\left(i_{HB} - i_{HC}\right) + C_{XH}\, i_{XA} \tag{A1a}$$

$$i_{DB} = \frac{1}{\sqrt{3}}\left(i_{HC} - i_{HA}\right) + C_{XH}\, i_{XB} \tag{A1b}$$

$$i_{DC} = \frac{1}{\sqrt{3}}\left(i_{HA} - i_{HB}\right) + C_{XH}\, i_{XC} \tag{A1c}$$

The same thinking applies to other connections. The resulting equations are given in Table A.1 (phase A only). The cited equations can be applied to either samples or phasors.

Table A.1. Differential currents for Yd connected transformers.

Connection	Phase A differential current
Yd30	$i_{DA} = \frac{1}{\sqrt{3}}\left(i_{HA} - i_{HC}\right) + C_{XH}\, i_{XA}$
Yd90	$i_{DA} = \frac{1}{\sqrt{3}}\left(i_{HB} - i_{HC}\right) + C_{XH}\, i_{XA}$
Yd150	$i_{DA} = \frac{1}{\sqrt{3}}\left(i_{HB} - i_{HA}\right) + C_{XH}\, i_{XA}$
Yd210	$i_{DA} = \frac{1}{\sqrt{3}}\left(i_{HC} - i_{HA}\right) + C_{XH}\, i_{XA}$
Yd270	$i_{DA} = \frac{1}{\sqrt{3}}\left(i_{HC} - i_{HB}\right) + C_{XH}\, i_{XA}$
Yd330	$i_{DA} = \frac{1}{\sqrt{3}}\left(i_{HA} - i_{HB}\right) + C_{XH}\, i_{XA}$

The differential currents given in Table A.1 are phase indexed. In the equations, however, the left-hand side index just repeats the index of the current i_X and is not tied so much with the index of the current i_H since as many as two phase currents from the H-side are involved. Therefore, the phase indexing for the differential current is formal only and counts when the comparison between the differential and restraining currents is made.

Certainly, all the Dy connected transformers are covered by Table A.1 with the side indices X and H switched.

In the Yy0 transformer there is no angular displacement between the primaries and secondaries and we obtain the following equations for the differential signals:

$$i_{DA} = \frac{1}{\sqrt{3}}[(i_{HA} - i_{HB}) + C_{XH}(i_{XA} - i_{XB})] \tag{A2a}$$

$$i_{DB} = \frac{1}{\sqrt{3}}[(i_{HB} - i_{HC}) + C_{XH}(i_{XB} - i_{XC})] \tag{A2b}$$

$$i_{DC} = \frac{1}{\sqrt{3}}[(i_{HC} - i_{HA}) + C_{XH}(i_{XC} - i_{XA})] \tag{A2c}$$

The differential currents (A2) are in secondary amperes of the CTs at the winding H.

Following the outlined reasoning Table A.2 has been created that gathers the differential current equations for Yy, Dd, Yz and Dz transformers.

Table A.2. Differential currents for various transformers.

Connection	Phase A differential current
Yy0	$i_{DA} = \frac{1}{\sqrt{3}}[(i_{HA} - i_{HB}) + C_{XH}(i_{XA} - i_{XB})]$
Yy180	$i_{DA} = \frac{1}{\sqrt{3}}[(i_{HA} - i_{HB}) + C_{XH}(i_{XB} - i_{XA})]$
Dd0	$i_{DA} = i_{HA} + C_{XH} i_{XA}$
Dd180	$i_{DA} = i_{HA} + C_{XH}(-i_{XA})$
Dz0	$i_{DA} = \frac{1}{\sqrt{3}}[(i_{HC} - i_{HA}) + C_{XH}(i_{XC} - i_{XA})]$
Dz180	$i_{DA} = \frac{1}{\sqrt{3}}[(i_{HC} - i_{HA}) + C_{XH}(i_{XA} - i_{XC})]$
Yz30	$i_{DA} = \frac{1}{\sqrt{3}}[(i_{HA} - i_{HC}) + C_{XH}(i_{XA} + i_{XN})]$
Yz330	$i_{DA} = \frac{1}{\sqrt{3}}[(i_{HA} - i_{HB}) + C_{XH}(i_{XA} + i_{XN})]$